If the left-hand point of the cable is fixed while the right-hand point moves with the displacement of a, b in the X and Y directions respectively, the boundary conditions are

$$x=0: \qquad v=0,\ w=0;$$

$$x=\ell: \qquad v=a\left(\cos\frac{2\pi t}{\tau}-1\right),\ w=b\sin\frac{2\pi t}{\tau} \tag{12b}$$

where τ is the period of the movement of the right-hand point. We can find the coefficient A in Eq. (11) by using the second of the Eqs. (10). In addition, we assume that vertical displacement of the right-hand point does not influence significantly the magnitude of A, i.e., $x=\ell$: $w=0$. Then Eq. (10) becomes

$$\begin{aligned}
&\epsilon d_c^2\left[\frac{12a}{\ell^3}(\ell-2x)\left(\cos\frac{2\pi t}{\tau}-1\right)\right.\\
&\left.-\frac{2q}{T_h}(\ell-2x)\,A\left(\cos\frac{2\pi t}{\tau}-1\right)\right]\\
&\times\left[\frac{q}{2T_h}(\ell-2x)+A(\ell-2x)\left(\cos\frac{2\pi t}{\tau}-1\right)\right]\\
&-\left[T_h+\frac{q^2}{8T_h}(\ell-2x)^2\right]2A\left(\cos\frac{2\pi t}{\tau}-1\right)\\
&+\frac{q^2}{8T}(\ell-2x)^2A\left(\cos\frac{2\pi t}{\tau}-1\right]\\
&+m\left[\frac{2\pi}{\tau}\right]^2A(x\ell-x^2)\cos\frac{2\pi t}{\tau}\\
&+\epsilon d_c^2\left[-\frac{3a}{\ell^2}(\ell-2x)^2\left(\cos\frac{2\pi t}{\tau}-1\right)\right.\\
&\left.+\frac{q}{2T_h}(\ell-2x)^2A\left(\cos\frac{2\pi t}{\tau}-1\right)\right]\\
&\times\left[-\frac{q}{T_h}-2a\left(\cos\frac{2\pi t}{\tau}-1\right)\right]\\
&+c\left[\frac{2\pi}{\tau}A(x\ell-x^2)\sin\frac{2\pi t}{\tau}\right]^2=0
\end{aligned} \tag{13}$$

to find the magnitude of coefficient A, we demand that Eq.(13) be valid for the point of the cable $x_1=\ell/8$ at the moment of time $t_1=\tau/4$. As a result

$$A=\frac{\epsilon d_c^2\dfrac{81}{8}\dfrac{a}{\ell}+\dfrac{27}{32}\epsilon d_c^2\dfrac{q^2}{T_h{}^2}\ell^2+\dfrac{27}{64}\dfrac{q^2}{T_h^2}+2T_h}{\dfrac{27}{16}\epsilon d_c^2\dfrac{q}{T_h}\ell^2-c\left(\dfrac{2\pi}{\tau}\right)^2\left(\dfrac{7\ell^2}{64}\right)^2}-\left[\left[\frac{\mathcal{A}}{\mathcal{B}}\right]^2-\frac{\mathcal{C}}{\mathcal{B}}\right]^{1/2} \tag{14}$$

where

$$\mathcal{A}=\epsilon d_c^2\frac{81}{8}\frac{a}{\ell}+\frac{27}{32}d_c^2\frac{q^2}{T_h^2}+\frac{27}{64}\frac{q^2}{T_h}\ell^2+2T_h$$

$$\mathcal{B}=\frac{27}{16}\epsilon d_c^2\frac{q}{T_h}\ell^2-c\left(\frac{2\pi}{\tau}\right)^2\left(\frac{7\ell^2}{64}\right)^2$$

$$\mathcal{C}=\frac{81}{16}\epsilon d_c^2\frac{q}{T_h}\frac{a}{\ell}$$

The increase in tension due to the movement of the right-hand point is

$$T_g=\epsilon d_c^2\left(-\frac{3a}{\ell}+A\frac{q}{2T_h}\ell^2\right)\left(\cos\frac{2\pi t}{\tau}-1\right)$$

where A is defined in Eq. (14).

Equation (14) seems to be rather complicated, but by using it, it is possible to analyze the influence of the cable's extensibility, length, and period of movement on tension in the cable, for example, when $\tau\rightarrow 0$, $A\rightarrow 0$. In this case the cable does not change its form. The movement of the right-hand point is possible only because of the cable's extension. Whenever $\epsilon\rightarrow\infty$, we will obtain the formula for an inextensive cable. Many assumptions which were made to obtain Eqs. (14) and (15) are obviously not valid. Consequently, an experiment was carried out to estimate their influence. The scheme of the experiment is shown in Fig. 3. In this experiment the tow-boat's movements and the cable tension were recorded simultaneously. A steel cable 100 m long with $d_c=17.5$mm was used. The difference between the cable tension measured and calculated was 5 to 7% of the maximum magnitude of tension.

Whenever the towing cable is made of two or more sections with different extensibilities, it is necessary to use average extensibility in Eqs. (14) and (15). When the cable consists of two parts, the average extensibility may be calculated using Hooke's law.

$$(\epsilon d_c^2)_{av}=(\ell_1+\ell_2)\,(\epsilon d_c^2)_1(\epsilon d_c^2)_2/[\ell_1(\epsilon d_c^2)_2+\ell_2(\epsilon d_c^2)_1]$$

where ℓ_i and $(\epsilon d_c^2)_i$ are length and extensibility of the ith part of the cable. This method is not exact, but its accuracy is reasonable enough for approximate calculations.

References

[1] Breslin, J.P., "Dynamic Forces Exerted by Oscillating Cables," *Journal of Hydronautics*, Vol. 8, Jan. 1974, pp. 19-31.

Flow Past an Anchored Slender Ship in Variable-Depth Shallow Water: An Extension

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Introduction

IN a previous paper, Plotkin[1] obtained the solution for the steady subcritical potential flow past a slender ship in shallow water in the presence of a slender bump. This work extended the constant-depth results of Tuck[2] by finding the second-order correction to the vertical force and pitching moment acting on the ship. It is the purpose of this Note to demonstrate that the solution technique of Ref. 1 is applicable to more general depth variations, and that changes in depth of the order of the slenderness parameter can lead to contributions to the forces of comparable magnitude to the constant-depth results.

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Description of Flowfield

Consider the steady subcritical potential flow of a uniform stream of speed U past a slender ship of length 2ℓ. A Cartesian coordinate system is located at midship with z measured upwards from the undisturbed free surface and x in the streamwise direction. The slender hull is described by $y=\epsilon f(x,z)$ where ϵ, the slenderness parameter, is a measure of the ratio of beam or draft to length. The shallow water assumption requires the depth also to be of $0(\epsilon)$ and the bottom is given by $z=-\epsilon h(x,y)$. The depth Froude number $F=U/(g\epsilon h)^{1/2}$ is taken to be of 0(1).

A perturbation velocity potential ϕ is defined which satisfies Laplace's equation and tends to zero at infinity. The fluid velocity is

$$\bar{q}=U\nabla(x+\phi) \tag{1}$$

ϕ must be determined such that the hull, bottom, and free surface are streamlines and the pressure is constant on the free surface.

Outer Expansion

In the outer region, far from the ship, the coordinates have the following orders of magnitude: $x,y==0(1)$, $z=0(\epsilon)$. The linearized shallow-water equation, valid for ϕ of $0(\epsilon)$, is given in Wehausen and Laitone[3] as

$$(1-F^2)\phi_{xx}+\phi_{yy}+(1+\phi_x)h_x/h+\phi_y h_y/h=0 \tag{2}$$

Consider a small variation from a constant-depth bottom. Let

$$\phi=\epsilon\phi_0(x,y),\quad h=h_0+\epsilon h_1(x,y) \tag{3}$$

and substitution into Eq. (2) yields

$$(1-F_0^2)\phi_{0_{xx}}+\phi_{0_{yy}}=-h_{1_x}/h_0 \tag{4}$$

where F_0 is based on the mean depth. Following Plotkin[1] the solution is written in terms of Green's function (source) distributions as

$$\phi_0=(2\pi\beta)^{-1}\Big\{\int_{-\infty}^{\infty}\mu(\xi)\log[(x-\xi)^2+\beta^2y^2]^{1/2}d\xi$$
$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h_{1_x}(\xi,\eta)\log[(x-\xi)^2+\beta^2(y-\eta)^2]^{1/2}d\xi d_\eta/h_0\Big\} \tag{5}$$

where $\beta^2=1-F_0^2$. The source strength $\mu(x)$ will be determined by matching.

Inner Expansion

In the inner region the coordinates have the following orders of magnitude: $x=0(1)$, y, $z=0(\epsilon)$. Inner variables Y and Z are introduced as

$$Y=y/\epsilon,\qquad Z=z/\epsilon \tag{6}$$

The mathematical problem for ϕ including terms of $0(\epsilon^2)$ is

$$\phi_{YY}+\phi_{ZZ}=0 \quad \text{in fluid domain} \tag{7a}$$

$$\phi_N=\epsilon^2 f_x(1+f_Z^2)^{-1/2} \quad \text{on } Y=f \tag{7b}$$

$$\phi_Z=0 \quad \text{on } Z=0 \text{ and } Z=-h_0 \tag{7c}$$

where N is the normal in the $Y-Z$ plane expressed in inner variables. The solution to Eqs. (7) can be written as

$$\phi=\epsilon f(x)+\epsilon^2 g(x)+\epsilon^2\Phi(Y,Z;x) \tag{8}$$

where f and g are arbitrary.

The hull area flux is $\epsilon^2 S_x(x)$ where $\epsilon^2 S(x)$ is the hull cross section beneath $Z=0$. If the hull is symmetric with respect to the plane $Y=0$, a suitable boundary condition for Φ is

$$\Phi\to u(x)\ |Y|+o(1)\ \text{as}\ |Y|\to\infty \tag{9}$$

where $u(x)$ is determined from conservation of mass as

$$u(x)=S_x(x)/2h_0 \tag{10}$$

Using the matching principle of Van Dyke,[4] the two-term inner expansion of the one-term outer expansion is equated to the one-term outer expansion of the two-term inner expansion. This results in

$$\mu(x)=S_x(x)/h_0,\qquad f(x)=\phi_0(x,0) \tag{11}$$

Inner Expansion of Pressure and Forces

The hydrodynamic pressure in the inner region, to $0(\epsilon)$, is given by the linearized Bernoulli equation as

$$p=-\rho U^2\epsilon\phi_{0_x}(x,0) \tag{12}$$

with ϕ_0 being given in Eq. (5). The vertical force, L, positive upwards, and the trim moment M, positive clockwise, are given by

$$L=\int_{-\ell}^{\ell}dx\,p(x)\,B(x),\quad M=-\int_{-\ell}^{\ell}xdx\,p(x)\,B(x) \tag{13}$$

where $B(x)$ is the width of the cross section at the waterline. Using Eq. (12) and an integration by parts, we get

$$L=\rho U^2\epsilon(2\pi\beta h_0)^{-1}\Big\{\int_{-\ell}^{\ell}\int_{-\infty}^{\infty}S_x(\xi)B_x(x)\log|x-\xi|\,d\xi dx$$
$$+\int_{-\ell}^{\ell}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}B(x)h_{1_x}(\xi,\eta)\frac{x-\xi}{(x-\xi)^2+\beta^2\eta^2}d\xi d\eta dx\Big\} \tag{14}$$

and

$$M=-\rho U^2\epsilon(2\pi\beta h_0)^{-1}\Big\{\int_{-\ell}^{\ell}\int_{-\infty}^{\infty}S_x(\xi)\,(xB)_x\log|x-\xi|\,d\xi dx$$
$$+\int_{-\ell}^{\ell}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}xB(x)h_{1_x}(\xi,\eta)\frac{x-\xi}{(x-\xi)^2+\beta^2\eta^2}d\xi d\eta dx\Big\} \tag{15}$$

The double intergral terms are the constant-depth results found by Tuck[2] and the triple integral terms are the contributions due to the depth variation.

For the special case of depth variation only in the streamwise direction, $h_1=h_1(x)$, particularly simple results for the variable-depth contribution to pressure, force and moment are obtained. First,

$$p=\rho U^2\epsilon(2\beta^2h_0)^{-1}\int_{-\infty}^{\infty}h_{1_x}(\xi)\ \text{sgn}\,(x-\xi)d\xi$$
$$=\rho U^2\epsilon(\beta^2h_0)^{-1}h_1(x) \tag{16}$$

since to recover the uniform stream at infinity, $h_1(\infty)=h_1(-\infty)=0$. Using this result and Eqs. (13), the force and moment become

$$L=\rho U^2\epsilon(\beta^2h_0)^{-1}\int_{-\ell}^{\ell}h_1(x)\,B(x)\,dx \tag{17a}$$

and

$$M=-\rho U^2\epsilon(\beta^2h_0)^{-1}\int_{-\ell}^{\ell}xh_1(x)\,B(x)\,dx \tag{17b}$$

Conclusions

A general solution has been obtained for the steady subcritical potential flow past a slender ship in shallow water with depth variations of the order of the slenderness parameter. The contributions to the pressure distribution, vertical force and trim moment from the depth variation are of the same magnitude as the constant-depth results of Tuck.[2] The pressure has been obtained to $0(\epsilon)$ and the force and moment to $0(\epsilon^2)$. For the special case of depth variation only in the streamwise direction, the pressure is seen to be proportional to the local depth deviation from the mean position of the bottom.

References

[1]Plotkin, A., "Flow Past an Anchored Slender Ship in Variable-Depth Shallow Water," *Journal of Hydronautics,* Vol. 9, 1975, pp. 103-106.

[2]Tuck, E. O., "Shallow-Water Flows Past Slender Bodies," *Journal of Fluid Mechanics,* Vol. 26, 1966, pp. 81-95.

[3]Wehausen, J.V. and Laitone, E.V., "Surface Waves," *Handbuck der Physik,* edited by S. Flugge, Vol. 9, Springer-Verlag, Berlin, 1960, pp. 446-778.

[4]Van Dyke, M., *Perturbation Methods in Fluid Mechanics,* Academic Press, New York, 1964.

Hydroelastic Ichthyoid Propulsion

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Introduction

THE use of undulating plates as a means of aquatic propulsion has been known for a long time. However, the development of this and of other propulsive methods has been discouraged by the outstanding simplicity and efficiency of the propeller. Nevertheless, in some cases, propulsion by undulating plates may have distinct advantages over the propeller, namely; a) in deep water applications, where sealing of rotating propeller shafts is difficult, and b) in muddy and weed-infested environments.

In a recent study, Botman[1] investigated several aspects of the feasibility of propulsion by undulating plates. In his experiments, Botman utilized a submerged cantilevered plate to drive a catamaran, the undulation of the plate being generated by a motor through mechanical links. He concluded that this is a feasible, but low efficiency, propulsive method which may find specialized application.

This Note presents an alternative means of generating the undulatory motion. This is by fluid flow through pipes attached on the plate, thus eliminating the mechanical links, and affording better control over the waveform of the undulating plate. This has been named "hydroelastic ichthyoid propulsion", as it is based on harnessing the energy associated with a *hydroelastic* instability, and the resulting motion of the plate is similar in form to that of swimming fish (*ichthys*).

Mechanism of Hydroelastic Ichthyoid Propulsion

It is well known that a cantilevered pipe conveying fluid becomes unstable by flexural flutter at sufficiently high flow velocities.[2,3] Herrmann and Nemat-Nasser[4] studied some aspects of the hydroelastic behavior of such systems by considering the dynamics of thin plates on which flexible pipes conveying fluid were attached. This system exhibits the same stability characteristics as a pipe by itself (although torsional instabilities may also occur for some configurations). As discussed by Paidoussis,[5] energy transfer from the fluid to the pipe or vice-versa occurs accordingly as the flow velocity is above or below the critical flow corresponding to the onset of flutter. At supercritical flow velocities energy is transferred from the flowing fluid to the pipe. This results in amplifying small free motions of the system. The ensuing limit-cycle motion is essentially in the second mode of the system, but with a nonstationary waveform involving a downstream propagating wave with increasing amplitude. This resembles the anguilliform mode of swimming of slender marine animals.[6]

It occurred to the author that a thin plate with attached flexible pipes, or a moulded composite structure, conveying fluid supplied by a pump, and oscillating in a (supercritical) limit cycle could well be an alternative means of propulsion to that of a mechanically driven plate. As has been shown by Lighthill,[6] the feasibility of anguilliform swimming of marine animals depends vitally on a downstream propagating wave. This in fact should have a phase velocity higher than the forward swimming speed. Therein lies the main advantage of the proposed propulsion mechanism: a downstream propagating wave is an inherent characteristic of the undulatory motion of the plate.

Model Experiments

A limited experimental program was undertaken to test the feasibility of this method of propulsion. A simple catamaran structure was built, with a ½ hp motor-pump unit supported between the floats, and powered by an overhead electric line, as shown in Fig. 1. The plate was typically 6-15 cm wide, 25-60 cm long, and with thickness $t \sim 0.25$ mm. Two tygon pipes (0.635 cm diam, 1.59 mm wall thickness) were clipped symmetrically on either side of the plate, as shown in Fig. 2, and the flow was divided equally between them by a specially made adaptor. The experiments were conducted in a tank of 91.5 cm×91.5 cm (3 ft×3 ft) cross-section and 15 m long. Spoilers at the rear of the catamaran controlled the forward speed of the craft.

Propulsion in such an arrangement occurs even without undulation of the plate, simply by the change in momentum of the internal flow. Thus the experiments were confined to deciding whether propulsion *with* undulatory motion could be more efficient than without. Clearly, if this were shown not to be the case, hydroelastic ichthyoid propulsion would be proven a failure. To this end, for each set of experimental parameters, the forward speed was measured with the plate undulating and not undulating, the latter being achieved by adding a stiffener to the upper edge of the plate (shaped so that the increase in drag was negligible).

The experimental procedure was as follows: the catamaran was held, and the power turned on at a specific setting of flow through the pipes. The catamaran was then released, and it moved forward. Allowing about 4 m for a constant speed condition to be reached, the motion was timed over the next 8.5 m, establishing a value of U. Approximate measurements of the phase velocity and frequency could be made by direct observation of the plate over a certain distance.

Observations and Results

There was no difficulty in achieving propulsion, typically with a forward speed $U \sim 1$ m/sec. The undulation of the plate was as expected, and as observed in previously conducted experiments in a water tunnel, a backward propagating wave was clearly present with downstream-increasing amplitude. Typically, the undulation frequency ω was 15 rad/sec and the wavelength was 0.6ℓ, ℓ being the length of the plate, so that the reduced frequency, $\sigma = \omega\ell/2U \sim 5$.

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